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Reply to "Comment on 'Chiral suppression of scalar glueball decay' "

Michael Chanowitz[1]

[1]*Ernest Orlando Lawrence Berkeley National Laboratory, University of California, Berkeley, CA 94720*

In [1] I observed that the amplitude for spin zero glueball decay is proportional to the quark mass, $\mathcal{M}(G_0 \to \overline{q}q) \propto m_q$, to all orders in perturbation theory, so that the ratio $\Gamma(G_0 \to \overline{u}u + \overline{d}d)/\Gamma(G_0 \to \overline{s}s)$ is calculable and small, even though the individual rates are not perturbatively calculable because of soft t and u channel quark exchanges. I noted that if hadronization of $G_0 \to \overline{q}q$ is an important mechanism for $G_0 \to \pi\pi$ and $G_0 \to \overline{K}K$, then $\Gamma(G_0 \to \pi\pi)$ is much smaller than $\Gamma(G_0 \to \overline{K}K)$, explaining a previous LQCD result[2] and supporting identification of $f_0(1710)$ with G_0. A more robust consequence, emphasized in [3], is that mixing of G_0 with $\overline{u}u + \overline{d}d$ (and perhaps also $\overline{s}s$) mesons is suppressed, so that the scalar (and pseudoscalar) may be the purest glueballs. In both [1] and [3] I emphasized the necessity to verify the existence and consequences of chiral suppression by a reliable nonperturbative method, which today can only be LQCD.

Chao *et al.* agree that $G_0 \to \overline{q}q$ is chirally suppressed but propose that $G_0 \to \overline{q}q\overline{q}q$, which is not chirally suppressed, is the dominant mechanism for $G_0 \to \pi\pi$. In the preceding Comment[4] and in a previous paper[5] they exhibit an $\mathrm{O}(\alpha_S)$ amplitude for the exclusive process $G_0 \to \pi\pi$ using light cone wave functions. Since pQCD for exclusive processes converges much more slowly than inclusive pQCD[6], the estimate is not quantitatively reliable at the experimentally interesting scale, $m_G = 1.7$ GeV, where even the applicability of ordinary inclusive pQCD is marginal. While the $\overline{q}q\overline{q}q$ mechanism might indeed dilute or remove chiral suppression of $G_0 \to \pi\pi$, it is not possible to decide, since the magnitude of neither the $\overline{q}q$ nor $\overline{q}q\overline{q}q$ contributions are reliably calculable.

Comparing the amplitudes for $\mathcal{M}(G_0 \to \overline{q}q)$ and $\mathcal{M}(G_0 \to \overline{q}q\overline{q}q \to \pi\pi)$ in [1] and [4, 5] it appears that both begin at first order in α_S, but this impression is misleading. It is easy to see that $\mathcal{M}(G_0 \to \overline{q}q\overline{q}q \to \pi\pi)$ vanishes in the chiral limit at $\mathrm{O}(\alpha_S)$ for on-shell constituent gluons. The $\overline{q}q\overline{q}q$ mechanism requires the quark from one gluon to combine with the antiquark from the other gluon to form a color singlet pion. But G_0 cms (center of mass) kinematics then requires both quarks to have the same energy fraction, $x = 2E_q/m_G$ and both antiquarks to have fraction $1-x$, with $m_\pi^2 = x(1-x)m_G^2$. One of the q or $\overline{q}$ constituents of each pion is then moving in the opposite direction to the pion in the G_0 cms. Boosting to an infinite momentum frame, one constituent is then at $x = 1$ and the other at $x = 0$, where the wave function vanishes. In the chiral limit, $m_\pi = 0$, this is already apparent in the G_0 cms. Since confining dynamics may put the gluons off-shell of order Λ_{QCD}, the amplitude does not actually vanish but is suppressed of order $\mathrm{O}(\Lambda_{\mathrm{QCD}}/m_G)$.

In the revised Comment the authors have responded to this observation with the added stipulation that the G_0 constituent gluons are maximally off-shell, of order m_G. Although this requirement was not imposed in [5], the result is apparently unchanged. Certainly one consequence is that f_g, the effective G_0gg coupling, cannot be identified with the corresponding coupling f_0 in [1] as is claimed in [4, 5], but reflects the off-shell tail of the G_0 wave function or implicitly contains a factor α_S at the hard scale m_G reflecting hard $gg \to g^*g^*$ scattering to push the gluons maximally off-shell. Alternatively, hard scattering of $\overline{q}q\overline{q}q$ can align the quarks suitably with the final state pions, with the amplitude then explicitly of order $\mathrm{O}(\alpha_S^2)$.

The relative magnitude of the $\overline{q}q$ and $\overline{q}q\overline{q}q$ mechanisms for $G_0 \to \pi\pi$ is not obvious. For the $\overline{q}q$ mechanism we do not know the magnitude of $\mathcal{M}(G_0 \to \overline{q}q)$ because both $\alpha_S(Q)$ and the running mass $m_q(Q)$ are evaluated at a soft scale, $\mathrm{O}(\Lambda_{\mathrm{QCD}})$, and thus are not under perturbative control. In addition we do not know the hadronization rate from $\overline{q}q$ to $\pi\pi$ and $\overline{K}K$ compared to multi-meson final states. On the other hand, $\Gamma(G_0 \to \pi\pi)$ via the $\overline{q}q\overline{q}q$ mechanism cannot be reliably estimated and is additionally suppressed by the square of the coupling, $\alpha_S(Q)^2$, evaluated at the largest scale in the problem, $Q = m_G$. It is then important to stress the agreement, expressed in both [1, 3] and [4], on the most important point: reliable nonperturbative methods are needed to determine whether $G_0 \to \pi\pi$ is chirally suppressed. We eagerly await LQCD "data" and data from BES II to clarify the issue.

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